SYNOPTIC: Orbit and Position Determination for Mars Orbiters and Landers,

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Orbit Determination; Viking Missions

Theme

Statistical estimates of the state of a spacecraft in orbit about Mars and of the location of a lander on the surface of Mars are generated assuming Earth-based tracking of both spacecraft. These statistics are presented parametrically for a range of orbital geometries, lander locations, and tracking schedules. The covariance matrices for the orbiter and lander states include the effects of model uncertainties in the Martian gravity field, the ephemeris of Mars, and tracking station locations. Eigenvector and eigenvalue analyses relate the tracking geometry to the orientation of the orbiter state error ellipses. The effects of mapping covariances forward in time are also presented.

Content

The Viking-type missions are designed to land two spacecraft on the surface of Mars and to have two spacecraft in orbit about Mars. Accurate knowledge of the location of the landers and the dynamic state of the orbiters will be required to achieve certain mission objectives. A statistical model is used to obtain the accuracy of estimating the lander and orbiter states from analysis of Earth-based range (light travel time) and Earth-based range rate (Doppler frequency) data. The model is developed under the assumptions that the estimator utilizes a weighted-least-squares (WLS) process where the data are unbiased, the data noise is uncorrelated in time, and the weighting matrix is the inverse of the data covariance. It is developed in the following manner: let x be the state vector at epoch to be estimated; y, the observable; p, an unknown vector which shall include dynamic and observation model uncertainties; and e, the noise on the data. To generate statistics on the estimator $\hat{x}$ of x it is assumed that p is a random vector with mean zero $[E(p) = 0]$ and covariance $\text{cov}(p) = P_p$. Further, it is assumed that $E(e) = 0$, $\text{cov}(e) = P_e$, and $E(pe^T) = 0$, where superscript T indicates the transposed array.

The equation relating the observations to the parameters is

$$y = Ax + Cp + e \tag{1}$$

The WLS estimator

$$\hat{x} = (A^T P_e^{-1} A)^{-1} A^T P_e^{-1} y \tag{2}$$

is one of the unbiased estimators of x since $E(\hat{x}) = x$. This estimator does not account for known model errors and, consequently, is not optimal but is characteristic of processes currently in use for orbit determination. The covariance of $\hat{x}$, $P_x = E[(\hat{x} - x)(\hat{x} - x)^T]$ will reduce to

$$P_{\hat{x}} = (A^T P_e^{-1} A)^{-1} + (A^T P_e^{-1} A)^{-1} A^T P_e^{-1} \times A^T P_e^{-1} C P_p C^T P_e^{-1} A (A^T P_e^{-1} A)^{-1} \tag{3}$$

with the use of Eqs. (1) and (2) and the foregoing assumptions. A priori statistics on $\hat{x}$ can be combined with the above in the usual statistical manner for independent estimates.

Variants of Eq. (3) which account for a priori statistics on $\hat{x}$ and which map the statistics at epoch to other times are used, in conjunction with the a priori statistics in Table 1, to generate statistics on $\hat{x}$. Estimates of the states of the spacecraft are obtained in Cartesian form for the orbiter and cylindrical form for the lander. These cylindrical coordinates are distance off the Martian spin axis (r-spin), position parallel to the spin axis (Z), and longitude. Definition of the lander location in inertial space is completed by utilizing three additional parameters: the angles P and Q, which represent small deviations between the assumed and true Martian poles, and the Martian rotational period.

The accuracy of estimating the lander position was studied parametrically using various length tracking periods per day, total tracking spans, lander locations, and initial epochs. Results were not significantly affected by the lander latitude over the range of interest from 30° south to 30° north. Figure 1 shows typical results for a lander at zero latitude, for various tracking intervals starting on February 14, 1974. The 30-day case actually has 5-day tracking intervals at the beginning and end of the 30 days and 20 days in the middle without data. The daily tracking intervals are centered about the time that the Earth crosses the lander meridian. The solid curves are generated using Eq. (3) with the a priori uncertainty for the lander parameters and the appropriate model errors. The dashed curves were generated using just the first term in Eq. (3) and correspond to the statistics which would result if there were no model errors. The model errors which contribute the most to the final covariance are the ephemeris errors; if they could be eliminated, the accuracy of determination of longitude, r-spin, P, and Q could be in the 10-m range, depending on the tracking schedule. The Z component is the largest contributor to the total error in position. The accuracy of estimating this parameter is a strong function of the daily tracking interval varying from 90 km to 20 km as the tracking interval varies from 2 hr to half a Martian day.

For the orbiters the period, which must be nearly synchronous with the rotation period of Mars for reconnaissance purposes, is 24.6 hr and yields a semimajor axis of ~20,400 km. The nominal periapse altitude is determined from energy, telecommunications, and reconnaissance considerations to be 1400 km. The resulting very elliptical orbit has an eccentricity of 0.766. Because the orbit is established from an interplanetary approach trajectory in a nearly minimal energy fashion, the major flexibility remaining for mission design is a rotation of the orbit about the approach direction. The inclination i, which will be defined by the latitude of the landing site, can vary from about −20° to 60°. The com-

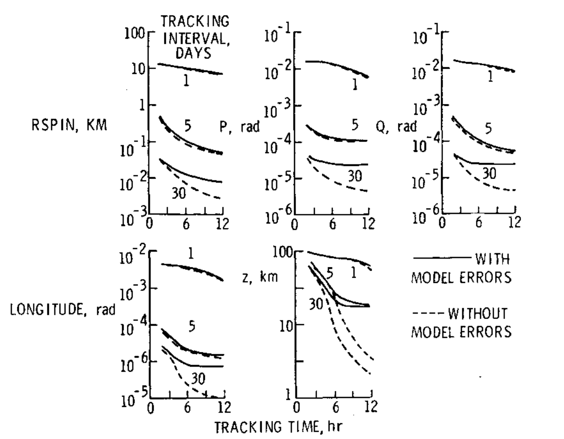

Fig. 1 Lander position standard deviations. Tracking starts on Feb. 14, 1974.

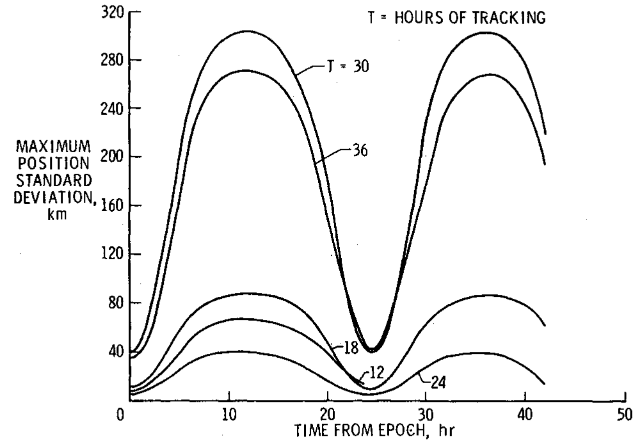

Fig. 2a Position standard deviation for epoch at periapse and orbital inclination of 30°.

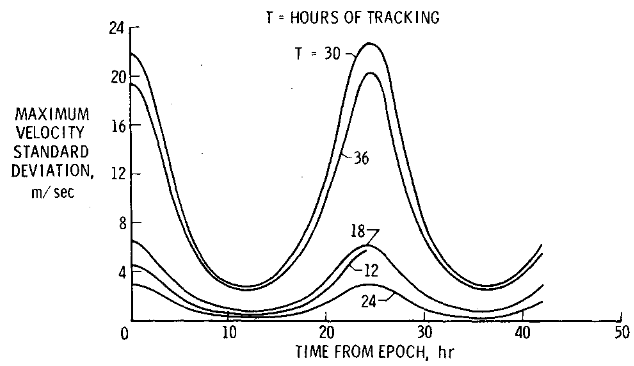

Fig. 2b Velocity standard deviation for epoch at periapse and orbital inclination of 30°.

paratively slow motion (~0.6°/day) of the Earth relative to the orbital plane introduces a serious orbit determination problem, because if there were no relative motion it would be impossible to determine all of the orbital elements from Doppler observations. In particular, with either Doppler or range data, the orientation of the orbit about the line of sight from the Earth to Mars is indeterminate. In the case of lunar satellites, the selenocentric parallax of widely spaced tracking stations, which can be four times the angular motion of the Earth in one orbit, can be used to eliminate the indeterminancy. However, for Martian satellites the corresponding parallax is only 10 arc sec, so this technique will not improve the situation, and it can be anticipated that the major un-

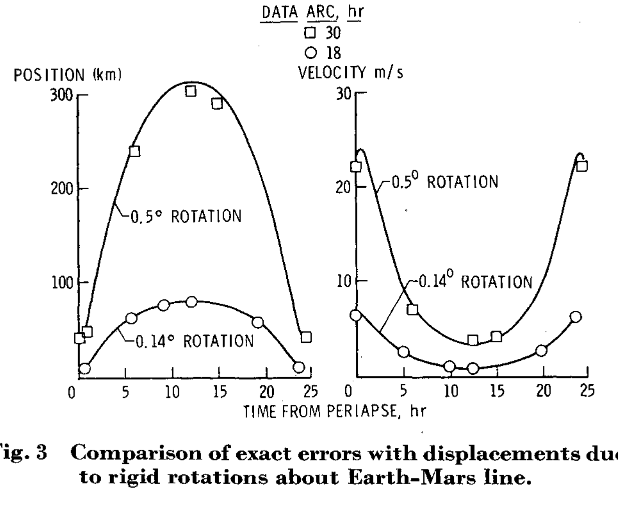

Fig. 3 Comparison of exact errors with displacements due to rigid rotations about Earth-Mars line.

Table 1 A priori Standard Deviations

Parameter	Standard deviation
Orbiter position	1000 km
Orbiter velocity	1 km/sec
Lander longitude	1 deg
Lander distance off spin axis	40 km
Lander Z component	100 km
Martian polar locations	1 deg
Martian rotational period	5×10^{-7} hr
Station distance off spin axis	0.0015 km
Tracking station Z component	0.025 km
Tracking station longitude	4.7×10^{-7} rad
Ephemeris position	5 km
Ephemeris velocity	5×10^{-7} km/sec
Astronomical unit	2 km
Martian GM	1.43 km^3/sec^2
Martian gravity field coefficients $\times 10^5$:	

n	m		n	m	
2	0	3.9	4	0	1.3
	1	2.2		1	0.41
	2	1.1		2	0.097
3	0	2.0		3	0.026
	1	0.83		4	0.009
	2	0.26			
	3	0.11			

certainty for Martian satellite orbit determination will be in i.

A parametric study of the accuracy of estimating the orbiter state was performed for typical Viking geometries, considering $i = 30°$ and $60°$. Tracking intervals of 12, 18, 24, 30, and 36 hr were analyzed, and for each the epoch time was taken to be at four points in the orbit corresponding to periapse, periapse plus 6 hr, apoapse, and apoapse plus 6 hr. The statistics at epoch for a WLS process were generated using Eq. (3), assuming Doppler measurements at 10-min intervals with a random noise of 0.001 m/sec, and using all appropriate a priori standard deviations from Table 1. The epoch statistics were mapped to other points in the orbit to investigate the variation with position in orbit and mapping time.

For example, consider the 24-hr curves in Fig. 2. The data span the interval of times from epoch (periapse) from 0 to 24 hr (0.6 hr prior to periapse). The statistics at epoch are then mapped through the first orbit and most of the second orbit. The periodic nature of the results, with only a small amount of secular growth, tends to confirm the anticipated result that the major uncertainty is due to an uncertainty in i. Further confirmation is based on a calculation of the angle between the position (velocity) eigenvector corresponding to the maximum position (velocity) uncertainty eigenvalue and the normal to the plane defined by the position (velocity) vector and the Mars-Earth vector. If the total uncertainty were due to an infinitesimal rotation about the line of sight, these two angles would be theoretically zero. The observed variation, over nearly all cases investigated, was less than 2°. In the cases where the angles are small, the ratio of the maximum to the intermediate standard deviation is usually greater than 25. Thus, these cases are characterized by a unidirectional, nearly degenerate error ellipsoid. For a small rigid rotation θ of the orbital plane about the Earth-Mars line, the expected change in position (velocity) is $\theta r \sin R$ ($\theta \nu \sin V$), where r is the radial distance of the satellite from Mars and R is the angle between the Earth-Mars line and the position vector (ν is the velocity relative to Mars, and V is the angle between the Earth-Mars line and the velocity vector). A θ of 1° produces a position displacement of ~630 km at apoapse. Figure 3 presents comparisons in which the symbols correspond to the points generated in the error analysis that were used to plot part of Fig. 2, and the curves correspond to the deviations calculated for θ's of 0.5° and 0.14°.

The agreement in trend and magnitude demonstrates that the major part of the error is plane orientation. The two maxima near periapse on the 0.5° curve (the curves span exactly one orbit; similar fluctuations occurring on the other three curves are masked by the symbols) occur because, for the orbital geometry investigated, the term $\sin V$ in the expression $\theta_\nu \sin V$ has a sharp minimum near periapse while the velocity ν has a flat maximum.

The strong dependence of the standard deviations on the length of the tracking interval T results from two strong opposing effects. First, as T increases, the knowledge of i improves, because additional data are being taken and the Earth is moving relative to the orbital plane. The Doppler data most sensitive to orbital plane perturbations are those near periapse. The opposing factor is the perturbation in the Doppler data due to gravity field uncertainties. Because the effect of the gravity coefficients drops off rapidly with radius from the planet, the effects of the nonspherical components of the gravity field are highly concentrated at periapse, and a small variation in the gravity field can produce the same order effect on the Doppler data as a rather large variation in i. The optimal interval appears to be slightly less than one orbit, starting just after periapse and ending just prior to the next periapse. Utilization of this tracking interval yields uncertainties within the design capability of the current Viking mission.

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Statistical estimates of the state of a spacecraft in orbit about Mars and of the location of a lander on the surface of Mars are generated assuming Earth-based tracking of both spacecraft. The statistics are presented parametrically for a range of orbital geometries, lander locations, and tracking schedules. The satellite geometries are highly eccentric, synchronous orbits consistent with Viking-type mission requirements. The total covariance matrix for the orbiter and lander states includes the effects of model uncertainties in the Martian gravity field, the ephemeris of Mars, and station locations. Eigenvector and eigenvalue analyses relate the tracking geometry to the orientation of the orbiter state error ellipses. The effects of mapping covariances forward in time are also presented.

THE Viking-type missions are designed to land two spacecraft on the surface of Mars and also to have two spacecraft in orbit about Mars. The orbiters will support the landers through reconnaissance of the landing area and will also act as relay stations for telemetry to the Earth if the direct link from the lander to the Earth is inoperable. There are various mission phases which require accurate knowledge of the location of the lander and the dynamical state of the orbiter. The purpose of this paper is to present some results of preliminary studies of the accuracy of determining the lander and orbiter states. Further, problem areas are indicated for which additional studies are required.

The results presented herein are restricted to the types of tracking data and orbital geometries expected during the Viking mission. The two types of tracking data that may be available are Earth-based range (light travel time) and range rate (Doppler) to all spacecraft. The Earth-based range rate data are assumed to be unbiased and to have a random noise of 1 mm/sec. Excluding periods of occultation, this type of data will be available essentially continuously from the orbiters; however, a more limited availability is expected from the landers due to power considerations. Although this paper has assumed a 1974 encounter date, the results should generally apply to the 1976 encounter as well.

The Earth-based range data appear to have limited operational or real-time utility after the spacecraft reaches Mars and, consequently, no consideration is given to range data in the analyses presented here. The reason for its ineffectiveness is the large uncertainties in the Martian ephemeris relative to the Earth. The anticipated radial ephemeris uncertainty during the mid-1970 time period is 5 km which is very large compared to the noise on the range data of 15 m. Simulations have shown that even if the ephemeris errors were negligible, the range data contribute little knowledge to the orbit determination as long as Doppler data are also available. However, as will be shown later, Doppler data are not very effective for completely determining the lander position, and preliminary studies indicate that range data could be very useful in completing the definition of the lander location if the ephemeris errors were somewhat smaller. It does not appear feasible to simultaneously determine the lander location and the Martian ephemeris because of the high correlation between these parameters for the short data arcs available during the early phases of the real-time mission. However, over longer data arcs the ephemeris and lander location effects become separable and the range data will then be a very effective data type.

The statistical model is based on the assumption that the estimator utilizes a weighted least-squares (WLS) process where the data are unbiased and uncorrelated and the weighting matrix is the inverse of the data covariance. The estimator does not account for known model errors and, consequently, is not optimal but is characteristic of processes currently in use for orbit determination. The equations are linearized and in the following discussion only the linearized equations are analyzed. Lower-case letters represent vectors, upper-case letters are matrices, and all dimensions are assumed consistent. Let x be the state vector at epoch to be estimated, y the observable, p an unknown vector which shall include dynamic and observation model uncertainties, and e the noise on the data. To generate statistics on the estimator $\hat{x}$ of x it is assumed that p is a random vector with mean zero $[E(p) = 0]$ and covariance cov $(p) = P_p$. Further, it is as-

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